THE NUCLEAR LIQUID-GAS PHASE TRANSITION

ISiS E900a Collaboration:
(Indiana, Texas A&M, Simon Fraser, Laval, Warsaw, Maryland, Argonne National Lab and Brookhaven National Lab)

8.0 GeV $\pi^-$ and $+^{197}$Au


Brookhaven AGS
Indiana Silicon Sphere ISiS
L. Beaulieu et al., PRC 64 064604 (2001).

Laboratory Invariant Cross Sections

Spectra

T. Lefort et al., PRC 64, 064603 (2001).
OUTLINE – THE EVIDENCE

Direct Experimental Observables

Equilibrium?
Multiplicities
Size Distribution
Breakup Density

Derived Quantities

Is it Statistical?
Extra Expansion
Time Scale
Caloric Curve Heat
Heat Capacity

Theory of Liquid-Gas Behavior

Percolation
Fisher Scaling

E/A* = Calorimetry
T = Double Isotope Ratios
LCP = H & He
IMF = Clusters Z = 3 –20
Z ≥20 = Heavy Residues/Fission Fragments
\[ P^m_n(E^*) = \frac{m!}{n!(m-n)!} p^n(1-p)^{m-n}. \]

\[ \langle n \rangle = mp \quad \text{and} \quad \sigma^2_n = \langle n \rangle (1-p). \]

<table>
<thead>
<tr>
<th>E*/A</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>MeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z_{5/6}</td>
<td>74.3</td>
<td>71.2</td>
<td>68.4</td>
<td>65.8</td>
<td>63.5</td>
<td>61.1</td>
<td>59.0</td>
<td>57.1</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>3.4</td>
<td>4.8</td>
<td>5.6</td>
<td>6.3</td>
<td>7</td>
<td>7.7</td>
<td>8.3</td>
<td>7.8</td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>-160</td>
<td>-204</td>
<td>-249</td>
<td>-295</td>
<td>-341</td>
<td>-383</td>
<td>-420</td>
<td>-451</td>
<td></td>
</tr>
</tbody>
</table>
Laboratory Invariant Cross Sections

Spectra

- SMM
- SMM + 0.5A MeV

L. Beaulieu et al., PRC 64 064604 (2001).
Expansion energy

Within the context of SMM calculations:

- Expansion energy increases slightly with $E^*/A$
- Thermally-induced origin
- Expanded nucleus at the break-up stage
- Central HI collisions: Expansion energy is larger
  Initial compression?

![Graph showing expansion energy vs. excitation energy at freeze-out for various experiments.](image)
velocity (v) and time (t)

\[ v_1 \text{ faster than } v_2 \]

\[ t_1 = t_2 \]

\[ t_2 \text{ longer than } t_1 \]

\[ v_1 = v_2 \]

\[ v_1 \approx v_2 \]

\[ t_1 \approx t_2 \]
$\sigma(Z) \propto Z^{-\tau}$

8 GeV/c $\pi^- + ^{197}$Au

$E_{\text{exp}}$ (A MeV)

$\tau$ (fm/c)

Excitation energy (A MeV)
J. Pochodzalla et al., PRL 75, 1040 (‘95)
A. Ruangma et al.,
PRC (submitted)
das Gupta et al, preprint (’02)
Breaking Probability

- Determined by the excitation energy deposited
- Infinite simple cubic lattice:
  - $p, \pi$ induced: eikonal approximation
  - $p_{\text{break}}$ proportional to path length through matter
- General relation between $p_{\text{break}}$ and $T$:
  
  $$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}} \Gamma \left[ \frac{3}{2}, 0, \frac{B}{T} \right]$$

  $\Gamma$ = generalized incomplete gamma function,
  $B$ = binding energy per nucleon
- Obtain $E^*$ from experiment

M. Berkenbusch et al., PRL 88, 22701-1 ('02)
Scaling Analysis

- Idea (Elliott et al.): If data follow scaling function

\[ N(Z,T) = Z^{-\tau} f \left[ \frac{T - T_c}{T_c} Z^\sigma \right] \]

with \( f(0) = 1 \) (think "exponential"), then we can use scaling plot to see if data cross the point \([0,1]\) -> critical events
Fisher Model [Physics 3, 255 (1967)]

J.B. Elliott et al., PRL 88, 42701-1 (2001)

\[ n_A = q_0 A^{-\tau} \exp \left( \frac{A \Delta \mu}{T} - \frac{C_0}{T} \right) \exp \left( \frac{E_{\text{coul}}}{T} \right) \]

- \( n_A \) = number of droplets of mass A
- \( q_0 \) = normalization constant
- \( \tau \) = topological critical exponent
  (Fisher = 2.1)
- \( \Delta \mu \) = \( \mu - \mu_L \), the actual and liquid chemical potentials
- \( C_0 \) = zero temperature surface energy coefficient (Nuclear matter \( \approx 16 \) MeV)
- \( \sigma \) = surface to volume dimensionality ratio
  (Nuclei = 2/3)

\[ \varepsilon = \frac{T_c - T}{T_c} \]

Control parameter that measures distance from the critical point
\[ \tau = 2.18 \pm 0.14 \]
\[ \sigma = 0.54 \pm 0.01 \]
\[ \Delta \mu = 0.06 \pm 0.03 \]
\[ C_0 = 18.3 \pm 0.5 \text{ MeV} \]
\[ T_c = 6.7 \pm 0.2 \text{ MeV} \]
\[ E^*_c = 3.8 \pm 0.3 \text{ MeV} \]
Figure 6. In reduced density-temperature space, the symbols show the path of the ISiS data; the line the coexistence curve of charged finite nuclear matter.

\[
\frac{\rho}{\rho_0} = \sum_{A=1}^{\infty} A \frac{n_A(T)}{\sum_{A=1}^{\infty} A n_A(T_c)}
\]
SUMMARY

THE VERDICT:

• Direct Observables
• Derived Quantities
• Scaling Theories (and Thermal Models)

ALL CONSISTENT WITH A NUCLEAR LIQUID-GAS PHASE TRANSITION

SENTENCING:

• Is the Phase Transition First- or Second-Order?
• What are the Critical Parameters?
• What do we Learn about the Nuclear Equation of State?

Research supported by the U.S. Department of Energy, the U.S. National Science Foundation and the National Sciences and Engineering Council of Canada